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Fair games of chance, such as Baccarat, are *exact*: the relevant elements of randomness are known, hence probability theory can be applied to deduce outcomes, notably the *house advantage*.

However, applied probability theory can prove unwieldy for computing the outcome of everyday gambling behaviours, such as betting systems. For example: *how exposed is the house if a winning streak causes a highly skewed playerbanker differential?* 

Simulation is a proven way to assess possible outcomes in such situations.

#### To start: Define the simulation parameters

Symbol	Meaning	Value	Domain
n <sub>max</sub>	maximum number of games	50	
n	game number		$1 \le n \le n_{max}$
р	probability of success for player in 1 game	0.44625	
b	probability of success for banker in 1 game	0.45860	
t	probability of tie in 1 game	0.09515	
$Z_n$	outcome on game n		$\{P, B, T\}$
$L_n$	current run length, game n		
$F_n$	final run length, game n		
$R_n$	bet of run follower on game n		$\{P,B\}$
X <sub>n</sub>	betting index		$1 \le \mathbf{x}_n$

### **Runs and their lengths**

A run may be a Player run or a Banker run.

### **Example of Player run**

We define a <u>player</u> run to be any contiguous sequence of outcomes beginning with a player win and containing no banker win. Ties may appear anywhere in a player run, but may not begin it. A run may be terminated by either a win for the Banker or by the last game (set at 50, the approximate number of plays in a shoe). We define the length of a player run to be simply the number of player wins in the run.

#### Examples of various run lengths

Game #, n	41	42	43	44	45	46	47	48	49	50	Run
											Length
Example 1	Р	Р	Ρ	Р	Р	В					5
Example 2	Р	Т	Т	Р	Р	Р	В				4
Example 3	Р	Т	т	т	В						1
		-	-								
Example 4	В	Р	Р	Т	Р	Р	Р	Р	Т	Р	7

### The run follower

We propose the existence of a player who follows runs, and call him the run follower (RF). *Management in Macau suggest this is the case as evidenced by a widening differential.* On the first game he bets on Player or Banker with probability 0.5 each. Thereafter, he bets on the same outcome (P or B) as he did on the previous game, unless there is a run of length two on the "other side", in which cases he switches to that side.

RF will stay on the side of a run even if the run breaks. He will only switch allegiance if a run of length 2 becomes evident "on the other side".

Example of following runs. Here, a follower run length of 5 is achieved – and the player would have *lost* that hand.

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18	0.65	В	-1	0	В	1	
19	0.97	т	-1	-1	В	2	
20	0.24	Р	1	0	В	2	
21	0.13	Р	2	0	В	1	
22	0.08	Р	3	0	Р	1	
23	0.30	Р	4	4	Р	2	
24	0.61	В	-1	-1	Р	3	
25	0.24	Р	1	0	Р	1	
26	0.32	Р	2	0	Р	2	
27	0.09	Р	3	0	Р	3	
28	0.95	Т	3	0	Р	4	
29	0.37	Р	4	4	Р	4	
30	0.79	В	-1	-1	Р	5	
31	0.40	Р	1	0	Р	1	
32	0.14	Р	2	2	Р	2	
33	0.63	В	-1	0	Р	3	
34	0.69	В	-2	0	Р	1	
35	0.91	т	-2	-2	В	1	

### **Betting patterns**

The amount wagered for a hand is a function of the length of "winning runs". We consider the following betting patterns: constant, linear, an actual observed pattern provided by a local establishment, and two hypothetical bet patterns (we created 2, but any patterns can be inserted).

### **Cumulative winnings/losses**

We may now compute cumulative house profits (losses) for each of the betting patterns. We could determine insights for wins/losses for individual hands.

Betting patterns examined. All patterns start with a bet of 100 'units'; the actuals follow an *S-shaped* betting pattern, levelling off at a run of length 10..

run length		Actuals1*	actuals2	actuals3	linear	constant
	1	100	100	100	100	100
	2	100	100	100	200	100
	3	133	443	1200	300	100
	4	315	1050	10500	400	100
	5	728	2427	24267	500	100
	6	1064	3547	35467	600	100
	7	1596	5320	53200	700	100
	8	2183	7277	72767	800	100
	9	2911	9703	97033	900	100
	10	3000	10000	100000	1000	100

Insights from the simulation (30,000 shoes of 50 hands each). Performance from a single shoe that had a max run length of 5.

30000	Constant	Linear	Actual1	Actual2	Actual3		
CURRENT SHOE							
Min bet	100	100	100	100	100		
Max bet	100	500	728	2427	24267		
Total bet (handle)	5000	8000	6190	10600	54367		
Net player payout	-625	-1425	-1104	-2689	-16592		
Min player payout	-100	-500	-728	-2427	-24267		
Max player payout	100	400	315	1050	10500		
RTP	-12.50%	-17.81%	-17.84%	-25.36%	-30.52%		

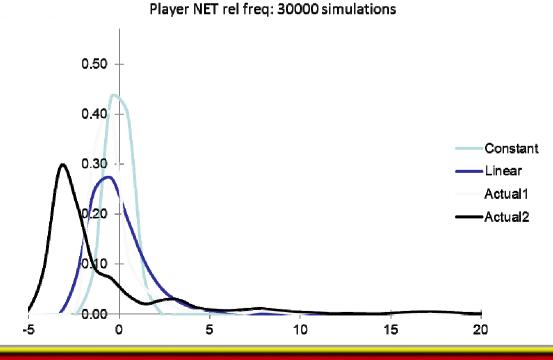
Outcome after 30,000 simulations (about a month's worth of play across 20 tables).

CUMULATIVE										
Prizes	-1587370	-3045755	-2901008	-6693614	-50996036					
Player Contribution	150000000	293756900	264054128	614046955	4521359306					
RTP	-1.06%	-1.04%	-1.10%	-1.09%	-1.13%					
	SUMMARY STATISTICS									
Min shoe revenue	-2635	-3535	-2635	-5334	-77278					
Max shoe revenue	2460	17010	36446	121346	1213072					
Median	-55	-375	-655	-2397	-18496					
Mean	-53	-102	-97	-223	-1700					
Std deviation	664	1559	2120	6813	67212					
Skewness	0	1	5	5	5					
Kurtosis	0	4	32	36	36					

#### This translates into 1,500,000 hands

	Player	Banker	Tie	TOTAL
Current outcomes	25	17	8	50
Cumulative outcomes	669438	687899	142663	1500000
Sim frequencies	0.44629	0.45860	0.095	1.000
Expected frequencies	0.44625	0.4586	0.095	1.000

Frequency distribution of net player payouts across the 30,000 simulations. Escalating bets means the player is more likely to lose; but if they win, they can win large amounts. Note how the mode shifts left as the differential increases.



Betting patterns can result in "outcome reversals" relative to constant betting. This could result in interesting behavioural ramifications.

	Non-co	nstant is	better		Constant is better				
	Linear	Actual1	Actual2	Actual3	Linear	Actual1	Actual2	Actual3	
#	1528	1093	1525	1957	3768	7529	8818	9217	
%	0.051	0.036	0.051	0.065	0.126	0.251	0.294	0.307	

### Conclusion

Regardless of differential (which can be used as a *product differentiation strategy*), the house advantage does not change; but the greater the differential, the greater the number of shoes that end with players losing and the more extreme are the payouts in the positive domain for players (the house paying out for a hand of play).

This simulation is therefore intended to help casino managers set betting limits that maximize total winnings while bearing in mind both the likelihood and magnitude of negative outcomes of increased differentials.

### **Questions?**

### Thank you!

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